Problem 1. (20 points)
The state of a system is given by the (unnormalized) wavefunction \( \Psi = e^{ia\theta - bx^2} \). The available space covers the range from \(-\infty\) to \(\infty\) in \(x\) and from 0 to \(2\pi\) in \(\phi\).
(a) Normalize the wavefunction.

(b) Find the expectation values for the following operators.
\[ \hat{p}_x \] (x-component of the linear momentum operator)
\[ \hat{l}_z \] (z-component of the angular momentum operator)

Problem 2. (20 points)
Consider rotational states of a diatomic molecule, e.g. \(H_2\). The energy difference between the energy levels with \(l = 1\) and \(l = 2\) of the molecule is \(E(2) - E(1) = 243.3\) cm\(^{-1}\).
(a) From these data determine the bond length in \(H_2\).

(b) What will be the energy difference between the same rotational levels for a half-deutrated (HD) and fully deuterated (D\(_2\)) molecule. Assume that the bond length does not change upon deuteration.